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Reliability Analysis of Counterfort Retaining Walls

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ABSTRACT: Traditionally, a constant factor of safety (usually 1.5) is adopted in the design of counterfort retaining walls against instability failure, regardless of the actual uncertainties in the various design variables. This constant factor of safety may not be able to quantify the uncertainties associated with the random variables. This paper presents the stability analysis of a typical counterfort retaining wall, accounting for uncertainties in the 'design variables' in the framework of probability theory. The first order reliability method (FORM), second order reliability method (SORM) and Monte Carlo Simulation (MCS) method are used in this study to evaluate the probability of failure associated with the various modes (both geotechnical and structural) of a typical counterfort retaining wall. Sensitivity analysis reveals that the angle of internal friction of the soil, is the most sensitive random variable, which affects all the modes of failure. Plots of reliability index and factor of safety are generated for critical modes of failure, where the constant factor of safety (as recommended in various design codes) is not able to get a desired reliability index/probability of failure.

Keywords: *Counterfort retaining walls, Factor of safety, Probability of failure, Reliability index*

1 INTRODUCTION

Retaining walls provide lateral support to vertical slopes of soil. They retain soil which would otherwise collapse into a more natural shape. They are common in highways and railway embankments, large constructions *etc*. The retained soil or backfill has a tendency to exert a lateral pressure against the retaining structure which is called active earth pressure. There are many different kinds of retaining walls, such as, cantilever, counterfort, butress *etc*. Generally cantilever walls are constructed of reinforced concrete for heights up to about 8 m. Above this height, the bending moments developed in the stem, heel slab and toe slab become very large and hence, thickness required for the stem, heel and toe slab becomes larger. In order to reduce the bending moment, transverse supports called *counterforts* are placed at regular intervals and hence it is called a counterfort retaining wall.

Traditionally, in the deterministic design of retaining walls a lumped 'factor of safety' approach is adopted to quantify the uncertainties in the design variables. The safety factor concept, however, has shortcomings as a measure of the relative reliability of geotechnical structures for different performance modes as the parameters are assigned single values when in fact they are uncertain (Fenton and Griffiths, 2008). Traditional deterministic theory may be used to model spatial variation, provided the spatially random soil can be represented by an equivalent uniform soil, which is assigned as an "effective" property. It has been shown for several geotechnical problems that the effective soil property can be based on an appropriate average value of the random soil property (Fenton and Griffiths, 2005).

Probabilistic methods are needed in geotechnical and structural engineering applications to account for variations in the design variables. This can be attributed to various factors such as complex geological variations, limited information of uncertain quality, poorer understanding of material behavior, and less controlled construction procedures and workmanship. Attempts have been made to account for these uncertainties in a more rational manner using probability theory by various geotechnical engineers (Whitman, 1984; 2000; Duncan, 2000; Phoon and Kulhawy, 1999; Christian, 2004). Christian *et al*. (1994), Chowdhury and Xu (1995), Liang *et al*. (1999) and others have described excellent examples of use of reliability analysis in geotechnical engineering. However, most of the previous studies in geotechnical reliability lack the rigour that is associated with structural reliability, even with the simplest treatment of the design variables as random variables (rather than random processes) and linearized limit state function (performance function). For the more general case of a nonlinear limit state function, the first order reliability method (FORM) based on Hasofer-Lind formulation is most appropriate as it gives consistent results. The results of the FORM can be refined by the second order reliability method (SORM), which takes into account the curvature of the failure function. Alternatively, Monte Carlo Simulation (MCS) can be used to evaluate directly the exact probability of failure.

Majority of the studies have focused on the reliability analysis of retaining walls for geotechnical failure modes (Hoeg and Muruga, 1974: Duncan, 2000; Fenton et al, 2005; Low 2005) and a very few have studies have focused on the reliability analysis of retaining walls for both geotechnical and structural failure modes (Sivakumar and Basha, 2008). This paper discusses the reliability analysis of a typical counterfort retaining wall for both geotechnical and structural modes of failure. The geotechnical failure modes, which need to be considered in the design of a counterfort retaining wall are (i) horizontal sliding along the base of the wall, (ii) the rotation about the toe of the wall (overturning), and (iii) bearing capacity failure of the soil beneath the wall, under the inclined and eccentric resultant load derived from the weight of the wall and the active earth thrust acting on the back of the wall. The structural failure modes which are to be considered are the bending moment and shear failures of the toe, heel, stem and counterfort slabs.

2 RELIABILITY ANALYSIS

The basic aim of structural reliability is to ascertain whether the desired strength (Resistance) R is much larger than the actual load (Strength) S throughout the useful life of the structure. Due to uncertainties in the determination of strength and loads, reliability can only be established in probabilistic terms, i.e., $P(R > S)$. These resultant variables (Resistance and Strength) are usually functions of several random variables $X_1, X_2, \ldots, X_n$.

The limit state function, associated with failure can be expressed as

$$M = R - S = g(X_1, X_2, \ldots, X_n) \qquad (1)$$

where M is referred to as the safety margin, which is a function $g(\boldsymbol{X})$ of the basic variables X_i. The condition $g(\boldsymbol{X}) < 0$ implies failure, while $g(\boldsymbol{X}) > 0$ implies stable behaviour. The boundary, defined by $g(\boldsymbol{X}) = 0$, separating the stable and unstable states is called the limit state boundary.

Mathematically the probability of failure $P_f = P(g(\boldsymbol{X}) < 0)$ can be simply defined as follows

$$p_f = \int \ldots \int_{g(X)<0} f_{\bar{X}}(x_1, x_2, \ldots, x_n) d_{x1} d_{x2} \ldots d_{xn} \qquad (2)$$

where $f_{\bar{X}}(x_1, x_2, \ldots, x_n)$ is the joint probability density function of all the basic variables. This joint probability density function may be visualized as a hypergeometrical space with unit volume, and P_f denotes the function of that volume which lies in the failure domain $g(X) < 0$. In terms of R and S, this can be easily visualized as shown in Figure 1.

The multiple integral of Equation 2 is, in general, very difficult to calculate. Thus, both analytical and simulation-based techniques have been developed for evaluating P_f efficiently and accurately. The commonly used analytical methods are FORM and SORM, whereas MCS is the easiest simulation based technique.

This integral equation (Equation 2) can be evaluated analytically using the Advanced First-Order second-moment (AFOSM) reliability method (Hasofer and Lind, 1974). The AFOSM method does not consider the tail behavior of the probability distribution of the random variables. To overcome this, the efficient FORM (Rackwitz and Fiessler 1978; Madsen *et al*. 1986) can be used to calculate reliability index:

The basic principles implied in FORM are as follows:

- The variables $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ are transformed by suitable transformations into a vector $\boldsymbol{U} = (U_1, U_2, \ldots, U_n)$ of standardized and independent (uncorrelated) normal variables.
- The limit state surface $g(\boldsymbol{U}) = 0$, formulated in this new space, is approximated by its tangent hyper plane at the point of smallest distance β to the origin as shown in Figure 2 for the case of two random variables. The distance β is called the reliability (or safety) index.

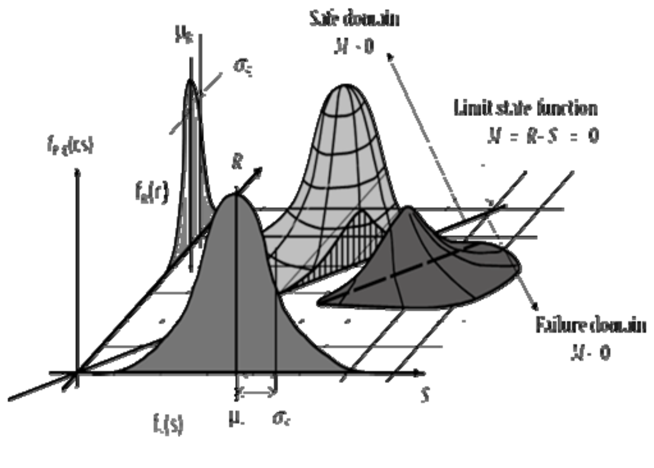

Figure 1. Failure probablity determination

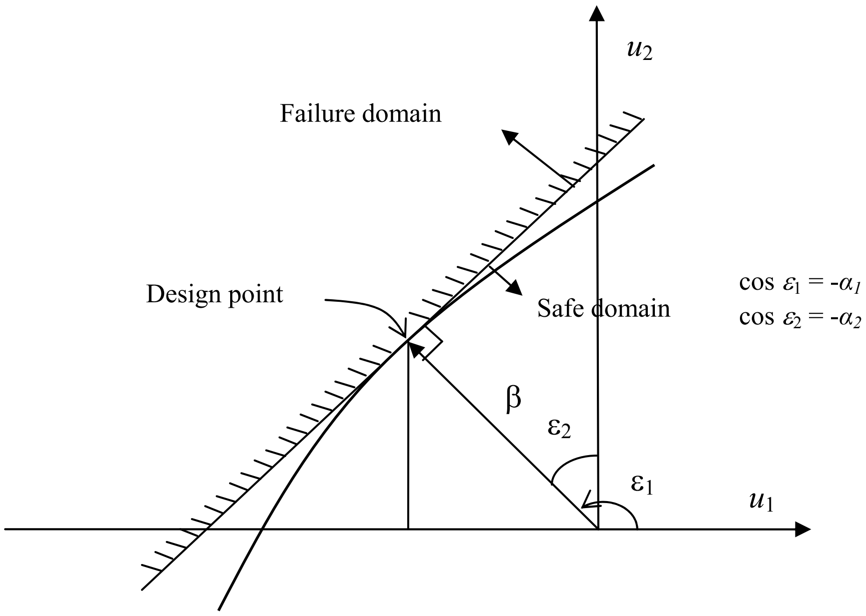

Figure 2. First order Reliability method for two variables

The point of smallest distance to the origin $\boldsymbol{u}^*$ is called the design point and is determined by an appropriate algorithm. The probability of failure is estimated by:

$$P_f = \Phi(-\beta) \tag{3}$$

where $\Phi(\bullet)$ = standard normal distribution function. The failure probability decreases with the increase of β. The influence of each random variable on the failure probability is expressed through the so-called sensitivity or importance factors α_i, which gives the directional cosines of the design point (Figure 2). The design value $\boldsymbol{x}^*$ for each influencing random variable X_i with distribution function F_{xi} is defined with the help of its sensitivity factor α_i and the reliability index:

$$x_i^* = F_{xi}^{-1}[\Phi(-\alpha_i\beta)] \tag{4}$$

For the load variables S, sensitivity vector for strength, $\alpha_S \le 0$ and for the resistance variables R, sensitivity vector for resistance, $\alpha_R \ge 0$.

For greater accuracy, when the curvature of the surface at the design point is significant, an improved method called SORM can be used (Breitung, 1984).

Equation 2 can be solved by an exact probabilistic method such as MCS method, where the probability of failure is computed from the joint probability distribution of the random variables associated with the loads and resistances. MCS is a simulation method that can be applied to many practical problems, allowing the direct consideration of any type of probability distribution for the random variables and it is able to compute the probability of failure with the desired precision.

The MCS method allows the determination of an estimate of the probability of failure, given by

$$p_f = \frac{1}{N}\sum_{i=1}^{n} I(X_1, X_2, ..., X_n) \tag{5}$$

where $I(X_1, X_2, \ldots, X_n)$ is a function defined by

$$I(X_1, X_2, \ldots, X_n) = \begin{cases} 1 & \text{if } g(X_1, X_2, \ldots, X_n) \le 0 \\ 0 & \text{if } g(X_1, X_2, \ldots, X_n) > 0 \end{cases} \tag{6}$$

Using MCS, an estimate of the probability of structural failure is obtained by

$$P_f = N_f / N \tag{7}$$

where N_f is the number of simulation cycles when $g(\boldsymbol{X}) < 0$ and N is the total number of simulation cycles. But the crude MCS has a drawback that as the value of P_f is very low (as encountered in structural reliability), the number of simulations required may be very large. This problem can be addressed by adopting "variance reduction techniques", such as importance sampling technique (Harbitz, 1986).

3 TYPICAL COUNTERFORT RETAINING WALLS

Guidelines for design of counterfort retaining walls (Pillai and Menon, 2009) are available for arriving at optimal proportions of typical counterfort retaining walls. The counterfort retaining wall adopted in the present study is based on such guidelines which will ensure the required deterministic factor of safety suggested by national codes of various countries. The objective of the present study is to check whether the proposed deterministic factor of safety is sufficient to quantify the uncertainties associated with the design variables.

The dimensions of the counterfort retaining walls in the present study are arrived at based on such design guidelines which aim to achieve the prescribed minimum factor of safety of '1.5' against various failure modes. The typical cross section of the selected counterfort retaining wall is shown in Figure 3. The backfill of the wall comprises of granular soil with a unit weight of 16 kN/m^3 and an angle of internal friction of backfill soil as 30°. Good soil for foundation is available at a depth of 1.5 m below the ground level with a safe bearing capacity of 170 kN/m^3. The thickness of the stem is linearly tapered from 300 mm at top to 600 mm at bottom. The coefficient of friction between the soil and concrete is 0.5. A shear key of size 0.3 m × 0.4 m is provided at a distance of 2.4 m from the toe as shown in Figure 3. The materials considered are 25 MPa concrete and reinforcing bars having an yield strength of 415 MPa. The counterforts having a thickness of 0.5 m are placed at a clear spacing of 3m.

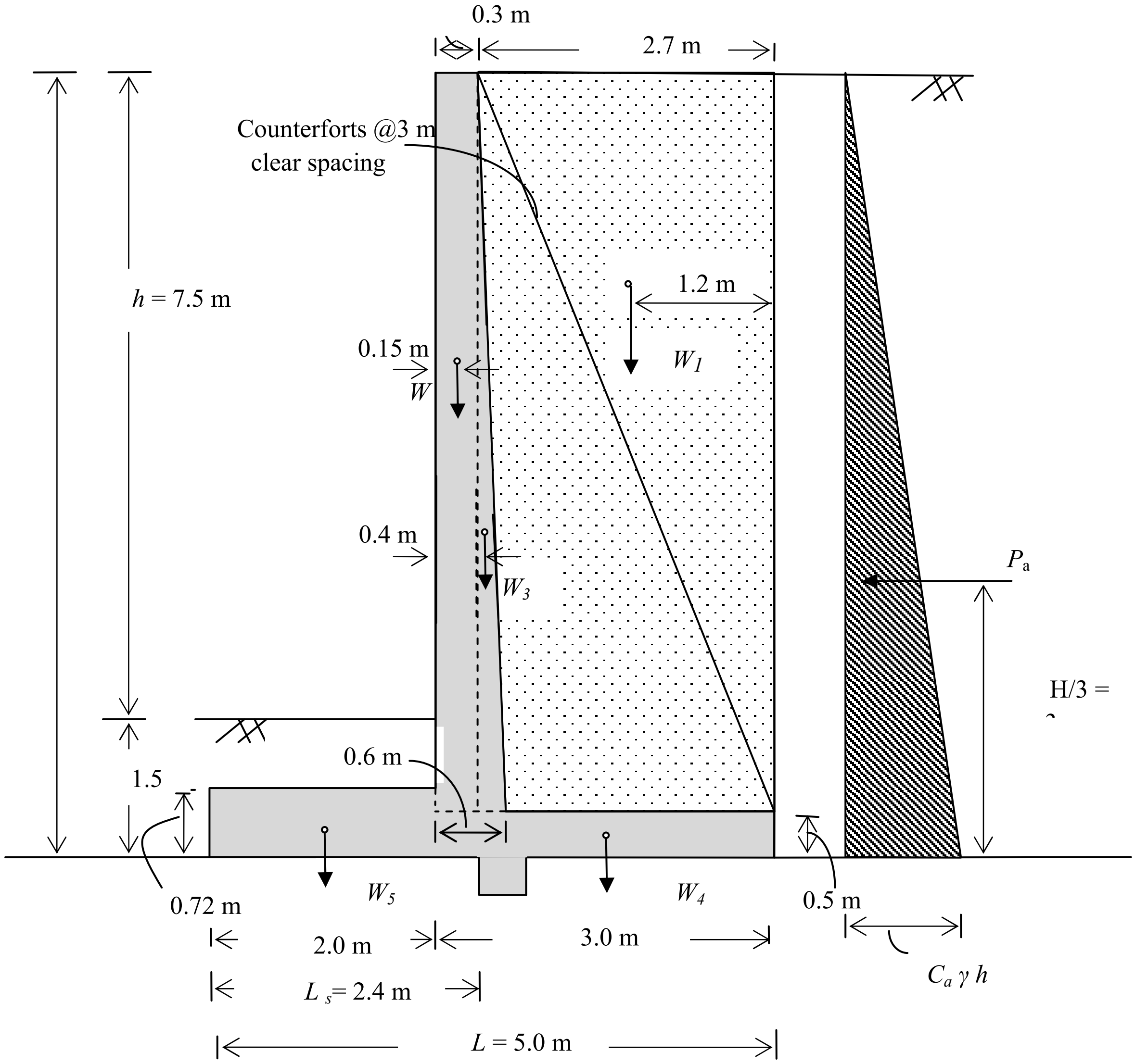

Figure 3. Typical Cross Section of the Counterfort Retaining Wall (Pillai and Menon, 2009)

Table 1. Statistical properties of random variables for geotechnical and structural failure modes.

Random variable	Mean	COV	Distribution	Reference
Unit weight of soil γ_s (X_1) (kN/m^3)	16	0.07	Normal	Castillo et al. (2004)
Angle of friction ϕ (X_2) (degree)	30	0.1	Normal	Castillo et al. (2004)
Coefficient of friction μ (X_3)	0.5	0.15	Normal	Castillo et al (2004)
Unit weight of concrete γ_c (X_4) (kN/m^3)	25	0.04	Normal	Ranganathan (1999)
Cohesion of soil at base c (X_5) (kN/m^2)	17	0.2	Normal	Sivakumar Babu and Basha(2008)
Angle of friction of soil below toe and heel ϕ_2 (X_6) (degree)	14	0.1	Normal	Sivakumar Babu and Basha (2008)

4 RELIABILITY ANALYSIS OF COUNTERFORT RETAINING WALL

The reliability analysis of the counterfort retaining wall is carried out with the statistical distribution of the variables and the values of coefficient of variation (COV) as shown in Table 1. All the variables are assumed to be normally distributed and the reliability analysis is carried out using the following methods: (i) FORM, (ii) SORM, (iii) MCS method, and (iv) MCS method with importance sampling.

4.1 *Geotechnical modes of failure*

The various modes of geotechnical failures which can occur on the counterfort retaining walls are Sliding, Overturning and Bearing failures.

4.1.1 *Sliding Failure Mode*

The factor of safety against sliding failure (FS_S) can be computed as

$$FS_S = \mu W / P_a \tag{8}$$

where W is the weight of the retaining wall and backfill over the heel, μ is the coefficient of friction at the interface of the concrete base slab and underlying soil and P_a is the total active earth pressure on the wall. Expressing μ, W and P_a in terms of the random variables X_1, X_2, X_3 and X_4 (Table 1), the performance function $g(\boldsymbol{X})$ (Equation 1) can be written as

$$\begin{aligned} g(X) &= R - S = \mu W - P_a \\ &= 21.675X_1X_3 + 6.765X_4X_3 - 40.5X_1\tan^2(45 - \frac{X_2}{2}) \end{aligned} \tag{9}$$

4.1.2 *Overturning Failure Mode*

The factor of safety against overturning can be computed as

$$FS_O = W(L - x_w) / (P_a x_p) \tag{10}$$

where x_w is the distance of line of action of W from the heel, x_p is the distance of its line of action from the base of the wall and L denotes the base width of the retaining wall.

The performance function for overturning mode of failure can be expressed as (Equation 1)

$$g(X) = 27.668X_1 + 18.595X_4 - 121.5X_1\tan^2(45 - X_2/2) \tag{11}$$

4.1.3 *Bearing Failure Mode*

Ultimate bearing capacity of a shallow foundation below the base slab of the retaining wall is given by

$$q_u = cN_cF_{cd}F_{ci} + qN_qF_{qd}F_{qi} + 0.5\nu BN_\nu F_{\nu d}F_{\nu i} \tag{12}$$

where q is the effective stress at the level of the bottom of the foundation $= \gamma_s h$ and h is the height of soil at the toe side of the wall in meters. F_{cd}, F_{qd}, $F_{\nu d}$ = depth factors; F_{ci}, F_{qi}, $F_{\nu i}$ = load inclination factors; and N_c, N_q, N_ν = bearing capacity factors. The maximum intensity of soil pressure at toe can be written as

$$q_{\max} = W/L\,(1 + 6[((W(L - x_w) + P_a x_p)/W) - (L/2)]/L \tag{13}$$

Factor of safety against bearing capacity failure can be defined as

$$FS_b = q_u / q_{\max} \tag{14}$$

The performance function for bearing mode of failure (Equation 1) can be expressed in terms of the variables $X_1, X_2, \ldots, X_6$ (Table 1) as

$$\begin{aligned} g(X) &= 0.6626X_5(\tan^2(45 + \frac{X_2}{2})e^{3.14\tan X_2} - 1)\cot X_2 \\ &- 29.16X_1\tan^2(45 - \frac{X_2}{2}) + 2.008X_1 - 1.5873X_4 \\ &+ 5.0X_1\tan X_2(\tan^2(45 + \frac{X_2}{2})e^{3.14\tan X_2} + 1) \\ &\times (1 - 22.87/X_6)^2 \end{aligned} \tag{15}$$

4.2 *Structural Modes of Failure*

Each panel of the stem and heel slab, between two adjacent counterforts are designed as a two way slabs fixed on three sides, and free on the fourth side (free edge). The toe slab is designed as a cantilever slab to resist the factored moments and shear forces. For all the cases, a load factor of 1.5 is used which ensures the necessary factor of safety by deterministic method. The cross section of the counterfort retaining wall, designed by deterministic method assuming the mean values of the random variables, with the reinforcement details on stem, toe slab and heel slab are shown in Figure 4. In addition to the variables considered for the geotechnical modes of failure, some more variables are considered for the structural modes of failure and their statistical distributions are shown in Table 3. The amount of steel, depth of the beam and clear cover is different for stem slab, toe slab, heel slab and counterfort slab, but their COV/standard deviation was kept the same values as shown in Table 3. The random variables such as amount of steel, depth of slab and clear cover for toe slab, heel slab and counterfort are presented in Table 4.

4.2.1 *Stem Moment Failure*

In the case of stem slabs in a counterfort retaining wall, cantilever action is limited to the bottom region (triangular region as shown in Figure 5) with the fixity at the junction of the stem with the base slab and elsewhere stem is treated as a continuous beam spanning between the counterforts (Pillai and Menon, 2009)

The factor of safety against stem moment is the ratio of the resisting moment (RM) to the overturning moment (OM), where the resisting moment can be computed as

$$RM = 0.87 f_y A_{st} (D - S_v - 0.42 x_u) \quad (16)$$

where x_u is the neutral axis depth and can be computed as

$$x_u = 0.87 f_y A_{st} / (0.36 f_{ck} b) \quad (17)$$

The terms f_y, A_{st}, D, S_v, f_{ck} are explained earlier and are shown in Table 3. In the Equation 17, b is the breadth of the section (here unit width of 1000 mm has been considered).

The maximum negative moment occurs in the stem at the counterfort location and the moment can be computed as

$$M_{u,-ve} = w_u L_{cc}^2 / 12, \textit{where } w_u = C_a \gamma_s h_1 \quad (18)$$

where, C_a is the active earth pressure coefficient, L_{cc} is the effective span between the counterforts

Table 2. Reliability index and probability of failure for geotechnical modes of failure.

Modes of failure	Deterministic factor of safety	Results of reliability analysis					
		FORM		SORM		MCS	MCS (importance sampling)
		β	P_f	β	P_f	P_f	P_f
Sliding	1.96	2.797	0.00257	2.781	0.00254	0.0025	0.0024
Overturning	2.19	8.451	1.4×10^{-15}	8.460	1.3×10^{-15}	–	1.35×10^{-15}
Bearing	3.00	3.001	0.0010	3.018	0.0012	0.0012	0.0013

Table 3. Statistical properties of random variables for structural failure modes.

Random variable	Mean	Std. dev.	Dist.	Reference
Concrete strength f_{ck} (X_7) (N/mm^2)	25	2.5	Normal	Ranganathan (1999)
Steel strength f_y (X_8) (N/mm^2)	415	20.75	Normal	Sivakumar Babu and Basha (2008)
Percentage steel A_{st} (X_9) (%)	0.018	0.009	Normal	Sivakumar Babu and Basha (2008)
Depth of the slab D (X_{10}) (m)	0.6	0.025	Normal	Ranganathan (1999)
Clear cover to the reinforcement S_v (X_{11}) (mm)	70	0.0025	Normal	Ranganathan (1999)
Thickness of counterfort D_c (X_{12}) (m)	0.5	0.025	Normal	Ranganathan (1999)
Thickness of stem at top s (X_{13}) (m)	0.3	0.025	Normal	Ranganathan (1999)

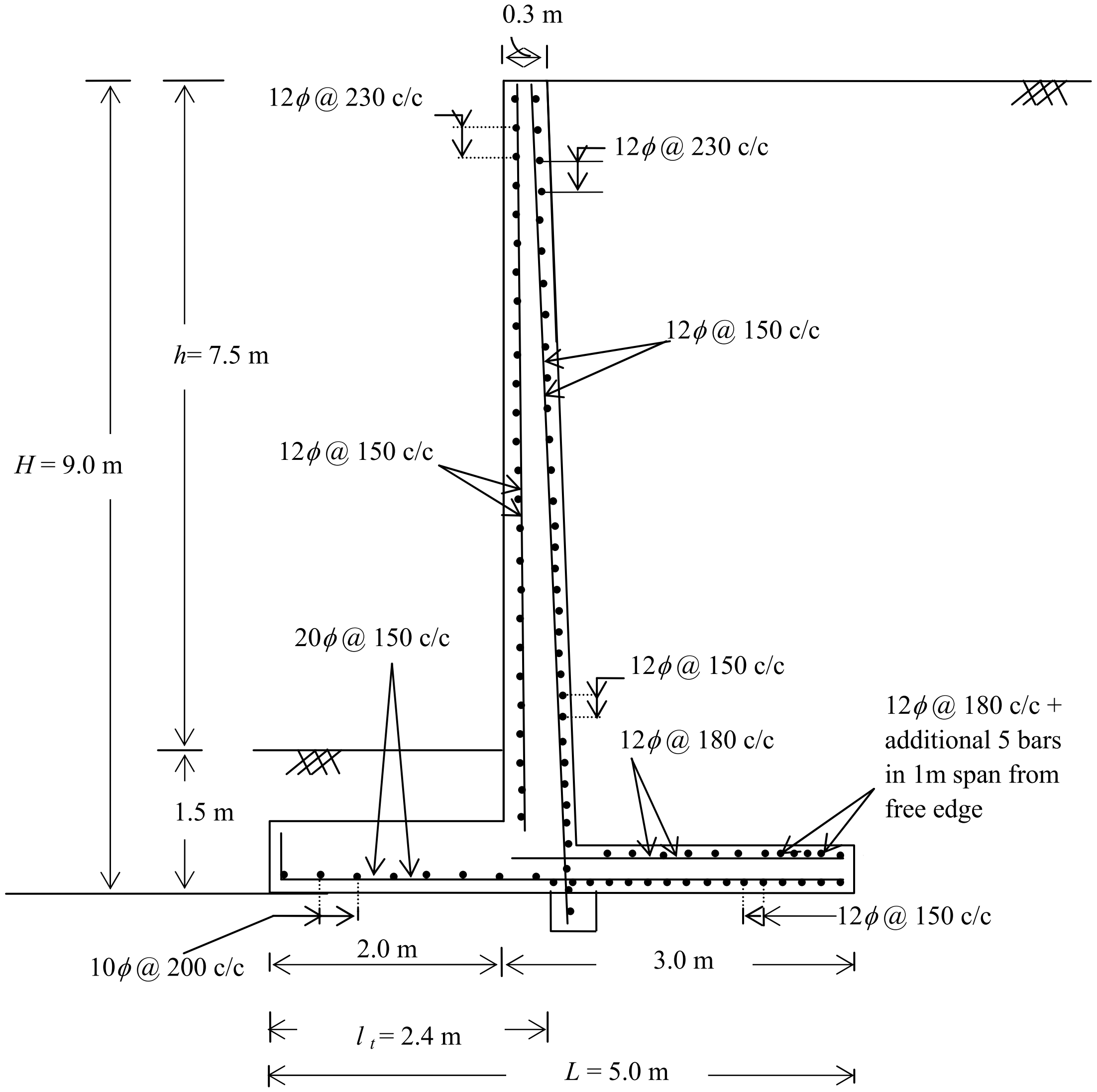

Figure 4. Reinforcement details on stem, toe slab and heel slab.

Table 4. Random variables for toe, heel and counterfort slabs.

Section	Amount of steel (%)	Depth of the section D (mm)	Clear cover S_v (mm)
Toe slab	0.32	720	75
Heel slab(–ve moment)	0.28	500	75
Heel slab(+ve moment)	0.17	500	75
Counterfort	0.35	2287	50

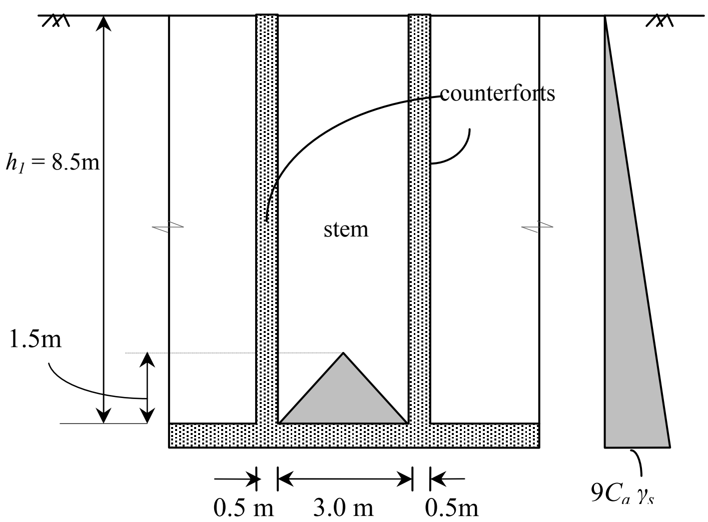

Figure 5. Loading considerations for simplified analysis of stem

Expressing the resisting and negative moments in terms of the random variables $X_1, X_2, ..., X_{13}$ defined in Tables 1, 3 and 4, the performance function for moment failure mode at stem can be written as (Equations 1, 16, 17 and 18)

$$g(X) = 0.87 X_7 X_8 - 8.8765 X_1 \tan^2(45 - \frac{X_2}{2}) \times (0.5 X_{10} + X_{13} - X_{11} - \frac{0.001105 X_8 X_9}{X_7}) \times 10^{-6} \quad (19)$$

4.2.2 *Stem Shear Failure*

Maximum shear force will occur at the end of the slab spanning between the counterforts and the critical section is considered at a distance 'd' from the junction of the stem and the counterfort, where d is the effective depth of the stem slab. The factor of safety against shear mode of failure is the ratio of the shear capacity of the section (RF) to the shear at the section due to the loads (SF).
The RF can be computed as

$$RF = \tau_c bd \quad (20)$$

where the magnitude of τ_c (shear capacity of a section) depends on the grade of concrete and percentage tension steel $p_t = 100 A_{st}/(bd)$. For the determination of τ_c, an empirical formula (Rangan,1972) can be used (Equation 21).

$$\tau_c = 0.85\sqrt{0.8 f_{ck}} \left(\frac{\sqrt{1+5\beta} - 1}{6\beta} \right) \quad (21)$$

where $\beta = 0.8 f_{ck}/(6.89 p_t)$ or 1 whichever is greater.
The SF for stem can be computed as

$$SF = w_u(L_c/2 - D + S_v) \quad (22)$$

Expressing RF and SF in terms of random variables $X_1, X_2, ..., X_{11}$ (Tables 1, 3 and 4), the performance function can be written as (Equations 1, 20, 21 and 22)

$$g(X) = 0.1124\left[\sqrt{1 + \frac{5.805 X_7 (X_{10} - X_{11})}{X_9}} - 1\right] \times \frac{X_9}{\sqrt{X_7}} - 8.5 X_1 \tan^2\left[45 - \frac{X_2}{2}\right] \quad (23)$$

For continuous beam action, consider a 1 m wide strip from the free edge of the heel. The average loading on the heel can be computed as $w_u = (P_3 + P_4)/2$. The maximum negative moment occurring in the heel slab at the counterfort location ($M_{u,-ve}$), maximum mid span moment ($M_{u,+ve}$) and the design shear force (V_u) can be computed as

$$M_{u,-ve} = W_u L_{cc}^2/12; \quad M_{u,+ve} = W_u L_{cc}^2/16; \quad V_u = W_u(L_{cc}/2 - D + S_v) \quad (24)$$

The resisting moment (RM) and the resisting shear force (RF) can be computed as depicted in the Equations 16, 17, 20 and 21. The performance function for negative and positive moments (Equation 1 and 24) can be expressed as

$$g(X) = 0.87 X_8 X_9\left[X_{10} - X_{11} - \frac{0.001105 X_8 X_9}{X_7}\right] \times 10^{-6} + 0.905 X_1 + 0.5188 X_4 - 22.804 X_1 \tan^2(45 - \frac{X_2}{2}) \quad (25)$$

$$g(X) = 0.87 X_8 X_9\left[X_{10} - X_{11} - \frac{0.001105 X_8 X_9}{X_7}\right] \times 10^{-6} + 0.679 X_1 + 0.384 X_4 - 17.104 X_1 \tan^2(45 - \frac{X_2}{2}) \quad (26)$$

The performance function for shear failure

(Equations 20 and 24) can be written as

$$g(X) = 0.1124\left[\sqrt{1 + \frac{5.805 X_7 (X_{10} - X_{11})}{X_9}} - 1\right] \times \frac{X_9}{\sqrt{X_7}} + 0.995 X_1 + 0.438 X_9 - 25.078 X_1 \tan^2\left[45 - \frac{X_2}{2}\right] \quad (27)$$

For cantilever beam action, the triangular loading on the heel slab [Figure 6(a)] with fixity at the face of the stem is considered. The bending moment due to loading can be evaluated as

$$M = 0.75 L_c [0.5 P_1 + 0.25(P_2 - P_1)] \quad (28)$$

where L_c is the clear spacing between the counterforts.

The performance function of heel slab for cantilever action (moment failure) (Equations 15 and 28) can be written as

$$g(X) = 0.87 X_8 X_9\left[X_{10} - X_{11} - \frac{0.001105 X_8 X_9}{X_7}\right] \times 10^{-6} - 0.966 X_1 + 1.858 X_4 + 3.791 X_1 \tan^2(45 - \frac{X_2}{2}) \quad (29)$$

4.2.3 *Toe Moment And Shear Failure Modes*

The net pressure acting on the toe slab can be computed by reducing the uniformly distributed self-weight of the toe slab $P_5 = D \times X_4$ from the gross pressure which varies linearly from q_{min} at the heel to q_{max} at the toe as shown in Figure 7.

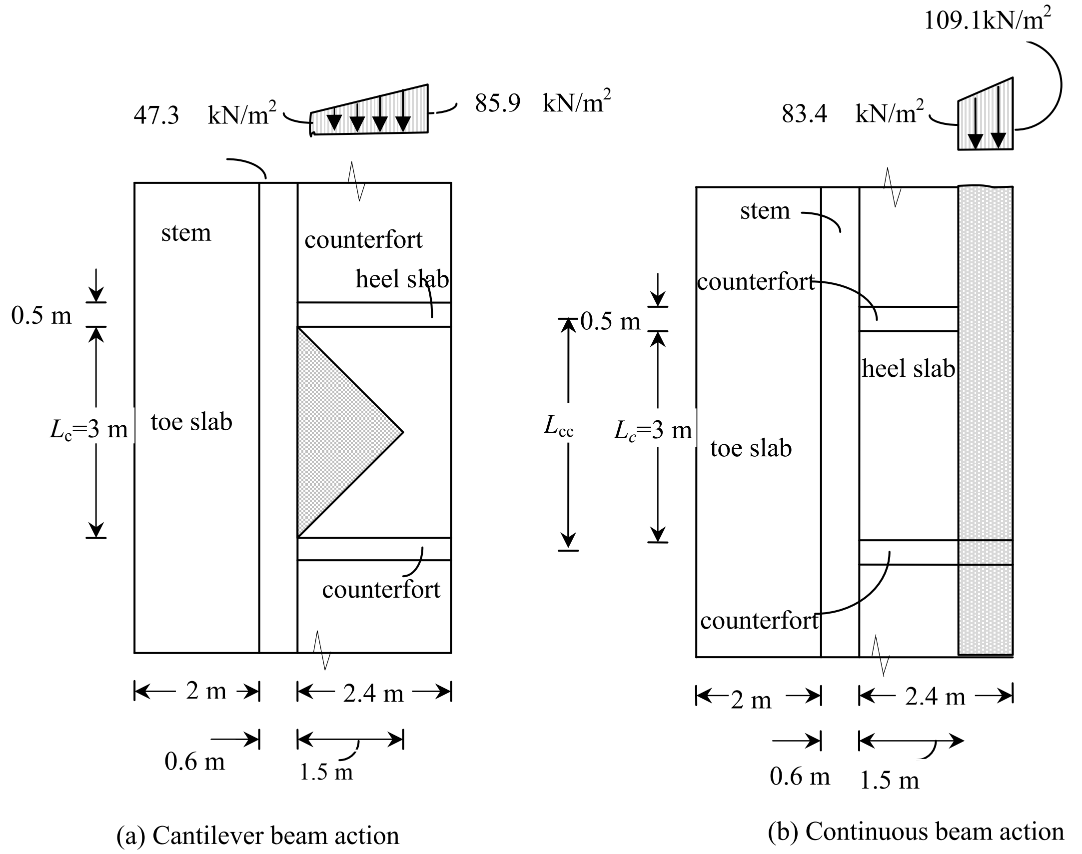

Figure 6. Loading considerations for simplified analysis of heel slab.

The critical section for bending moment is at the front face of the stem and the critical section for shear is at a distance d from the front face of the stem. The loading diagram on the toe slab is shown in Figure 7.

The pressure at the junction of the toe and the stem can be found by

$$P_t = q_{\min} + \frac{3(q_{\max} - q_{\min})}{5} \tag{30}$$

The bending moment and shear force can be computed from the pressure distribution diagram as shown in Figure 7. The $q_{\max}$ (Equation 13) and $q_{\min}$ corresponds to the maximum and minimum pressures developed on the base slab. The minimum pressure at the heel slab, $q_{\min}$ can be found by the equation,

$$q_{\min} = W\!/\!_L(1-6[((W(L-x_w)+P_a x_p)/W)-(L/2)]/L \tag{31}$$

The expressions for bending moment at the face of the stem and shear force at d distance from the front face of the stem are shown in the Equation 32.

$$v_u = 0.5(q_{\max} + P_t - 2P_5)(l_t - d);$$
$$M_u = 0.5 l_t^2 (P_t - P_5) + l_t^2 (q_{\max} - P_t)/3 \tag{32}$$

Expressing V_u and M_u in terms of random variables $X_1, X_2, \ldots, X_{12}$ (Tables 1, 3 and 4), the performance function for moment can be expressed as (Equations 16 and 32)

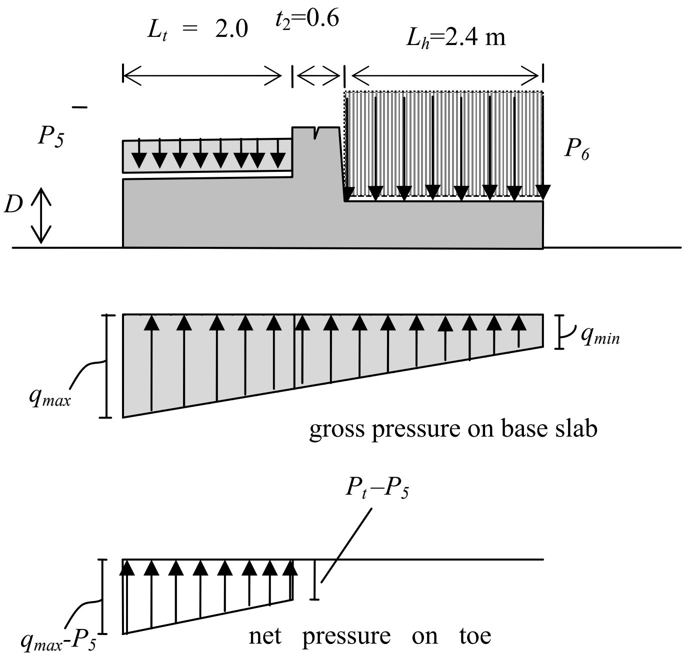

Figure 7. Pressure distribution on toe slab

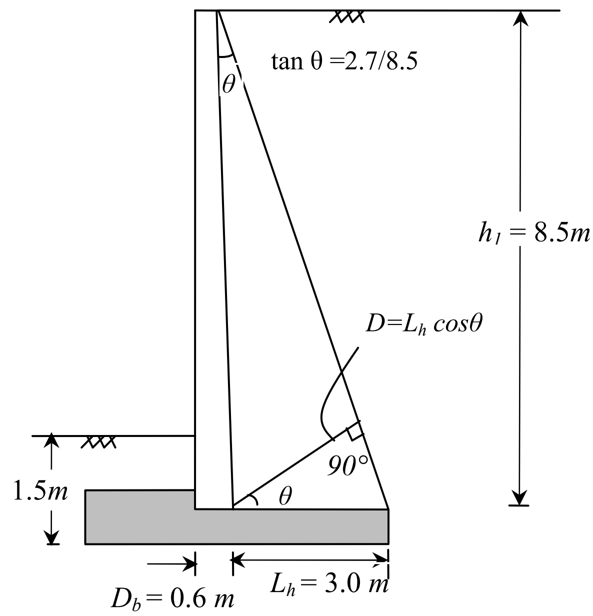

Figure 8. Depth consideration for analysis of counterfort

$$g(X) = 0.87X_8X_9\left[X_{10} - X_{11} - \frac{0.001105X_8X_9}{X_7}\right]\times 10^{-6} + 0.6479X_1 - 1.8514X_4 - 102.35X_1\tan^2(45 - \frac{X_2}{2})(3000 + X_{12})\times 10^{-3} \quad (33)$$

and performance function for shear can be expressed as (Equations 20, 21 and 32)

$$g(X) = 0.1124\left[\sqrt{1 + \frac{5.805X_7(X_{10} - X_{11})}{X_8}} - 1\right] \times\frac{X_8}{\sqrt{X_6}} - 0.5158X_1 - 0.8778X_4 - 17.496X_1\tan^2\left[45 - \frac{X_2}{2}\right](2000 - X_9 - X_{10})\times 10^{-3} \quad (34)$$

4.2.4 *Counterfort Moment And Shear Failure Mode*

The counterfort acts as a T-beam of varying cross section cantilevering out of base slab. For the counterfort retaining wall considered in this study, clear spacing of counterforts is 3 *m* and the thickness of each counterfort is 0.5 *m*. Hence the centre to centre spacing of counterforts (L_{cc}). The effective depth d can be calculated by deducting effective cover from D (as shown in Figure 8).

For the counterfort retaining wall considered in this paper, neutral axis lies in the flange. The lever arm for bending moment computation can be assumed as $0.9\,d$. The required amount of steel can be computed by equating total force of tension to total force of compression. The bending moment and shear force on the assumed section can be computed as

$$V_u = C_a \Upsilon_s h_1^2 L_{cc}/2; \quad M_u = C_a \Upsilon_s h_1^3 L_{cc}/6 \quad (35)$$

since the section is of varying depth, the net shear force at the section (*SF*) can be computed as

$$V_{u,net} = V_u - M_u \tan\theta / d \quad (36)$$

Expressing V_u and M_u in terms of random variables $X_1, X_2, \ldots, X_{12}$ (Tables 1, 3 and 4), the performance function (Equations 16 and 35) for bending moment can be expressed as

$$g(X) = 0.87X_8X_9\left[X_{10} - X_{11} - \frac{0.001105X_8X_9}{X_7}\right] \times 10^{-6} - 102.35X_1\tan^2(45 - \frac{X_2}{2})(3000 + X_{12})\times 10^{-3} \quad (37)$$

and performance function for shear can be expressed as (Equations 19, 20, 33 and 34)

$$g(X) = 0.1124\left[\sqrt{1 + \frac{5.805X_7(X_{10} - X_{11})}{X_9}} - 1\right] \times\frac{X_9}{\sqrt{X_7}} - 36.125X_1\left[45 - \frac{X_2}{2}\right](3 + X_{12}\times 10^{-3}) - 32.467X_1\tan^2\left[45 - \frac{X_2}{2}\right](3000 + X_{12})/(X_{10} - X_{11}) \quad (38)$$

5 RESULTS AND DISCUSSIONS

5.1. *Results And Discussion For Geotechnical Modes Of Failure*

The results of the reliability analysis for geotechnical modes of failure are summarized and shown in Table 2. Table 2 indicates that reliability/probability of failure obtained by FORM and SORM are comparable with the probability of failure generated by MCS. This may be attributed to the fact that the performance function has a distribution, which is close to the normal distribution

5.2 *Results And Discussion For Structural Modes Of Failure*

The results of the reliability analysis for the structural modes of failure for the counterfort retaining wall considered in this paper are summarized in Table 5. It is observed that the usual factor of safety of 1.5 (usually adopted in the design practice of many countries) is not sufficient to get a 'target' reliability index of 3, which is required to avoid any unforeseen structural and geotechnical modes of failure. Among the various modes of failure, mid span moment at heel (positive moment) is the most critical one. MCS is not able to catch probability of failure for some failure modes (stem moment, shear, toe shear, counterfort shear) as the probability of failure is very less and a very large number of simulations are required to capture the probability of failure which cannot be achieved by using ordinary computers In majority of the cases, results of FORM, SORM and MCS can be comparable, because the performance functions have a distribution, which is close to the normal distribution.

Table 5. Reliability index and probability of failure for structural modes of failure.

Modes of failure	Deterministic factor of safety	Results of reliability analysis					
		FORM		SORM		MCS	MCS (importance sampling)
		β	P_f	β	P_f	P_f	P_f
Stem moment	1.50	7.0653	8.0×10^{-23}	7.0476	9.1×10^{-23}	–	9.3×10^{-23}
Stem shear	1.50	9.2800	8.4×10^{-19}	9.2810	8.4×10^{-19}	–	8.5×10^{-19}
Heel moment(–ve)	1.78	5.1509	1.2×10^{-7}	5.1264	1.4×10^{-7}	–	1.4×10^{-7}
Heel moment(+ve)	1.50	2.5156	0.0059	2.5135	0.0059	0.0058	0.0059
Heel shear	1.50	2.5082	0.0021	2.8527	0.0021	0.0021	0.0022
Toe moment	1.50	4.1478	1.6×10^{-5}	4.1389	1.7×10^{-5}	–	1.7×10^{-5}
Toe shear	1.50	4.1640	1.5×10^{-9}	4.1669	1.5×10^{-9}	–	1.5×10^{-9}
Counter moment	1.50	3.4943	2.3×10^{-4}	3.4975	2.3×10^{-4}	2.3×10^{-4}	$2.\ \times10^{-4}$
Counter shear	1.50	4.3929	5.5×10^{-6}	4.402	5.3×10^{-4}	–	5.4×10^{-4}
Stem Moment	1.50	7.0653	8.0×10^{-23}	7.0476	9.1×10^{-23}	–	9.3×10^{-23}

6 SENSITIVITY ANALYSIS

Sensitivity analysis identifies the most sensitive random variables influencing the probability of failure. It is expressed in FORM analysis through the so-called 'sensitivity factor' α_i, which is measured in terms of the directional cosines of the position vector of the "design point", in the transformed U-space, as shown in Figure 2 for a case involving two random variables.

For load variables S, the value of α_S should be less than zero and for the resistance variables R, α_R should be greater than 0. The sensitivity vector for random variables considered in the geotechnical modes of failure are shown in Table 6 and for structural modes of failure are shown in Table 7. Sensitivity analysis reveals that angle of internal friction of the backfill soil (ϕ) is the most sensitive random variable affecting the reliability/probability of both geotechnical and structural failure modes. As shown in Table 6, angle of internal friction of backfill soil (ϕ) is the most sensitive random variable. For sliding failure, the most sensitive random variables are angle of internal friction of backfill soil (ϕ) and coefficient of friction (μ) in order. It is seen from Table 7 that angle of internal friction of the backfill soil (ϕ) is the most sensitive random variable, which affects all the structural modes of failure. From the reliability analysis of the counterfort retaining wall considered in the present study, it is observed that the geotechnical failures are more critical than the structural modes of failure. Among the geotechnical modes of failure, sliding of the counterfort retaining wall is the most critical one; whereas for structural modes of failure, negative moment at the heel slab is the critical one. As the factor of safety for sliding and overturning modes of failure for the counterfort retaining wall considered in the present study, is greater than 1.5 (usually adopted in design practices), it is desirable to plot the factor of safety (FS) versus reliability index (β) for sliding and overturning modes of failure.

Table 6. Sensitivity factors for geotechnical modes of failure.

Random variable $(X_i)\rightarrow$		$\gamma_s(X_1)$	$\phi(X_2)$	$\mu(X_3)$	$\gamma_c(X_4)$	$c(X_5)$	$\phi_1(X_{12})$
Sensitivity factor (α_i)	Sliding	0.0618	–0.8243	–0.5617	–0.0359	–	–
	Overturning	0.1287	–0.9883	–	–0.0814	–	–
	Bearing	0.0765	–0.8303	–	0.0159	–0.3958	0.3846

Table 7. Sensitivity factors for structural modes of failure

Random variable (X_i)	Sensitivity factor (α_i)								
	Stem moment	Stem shear	Heel moment (–ve)	Heel moment (+ve)	Heel shear	Toe moment	Toe shear	Counter moment	Counter shear
$\gamma_s(X_1)$	0.3906	0.4205	0.4158	0.4057	0.4278	0.4113	0.4693	0.4497	0.482
$\phi(X_2)$	–0.7146	–0.8079	–0.8096	–0.7864	–0.8288	–0.7596	–0.8011	–0.8031	–0.873
$\gamma_c(X_4)$	-	-	–0.0169	–0.0213	–0.0184	0.0342	0.0404	-	-
$f_{ck}(X_7)$	–0.0175	–0.0422	–0.0258	–0.0180	–0.0448	–0.0433	–0.0687	–0.0091	–0.0447
$f_y(X_8)$	–0.3822	-	–0.2430	–0.2851	-	–0.3808	-	–0.3769	-
$A_{st}(X_9)$	–0.0328	–0.0171	–0.0228	–0.0275	–0.0132	–0.0351	–0.0183	–0.0352	–0.0176
$D(X_{10})$	–0.0295	–0.4083	–0.3316	–0.3643	–0.3553	–0.3216	–0.3604	–0.082	0.0102
$S_v(X_{11})$	0.0387	0.0408	0.0322	0.0364	0.0355	0.0322	0.036	0.0082	–0.001
$D_c(X_{12})$	-	-	-	-	-	-	-	0.0509	0.0564
$s(X_{13})$	–0.2950	-	-	-	-	-	-	-	-

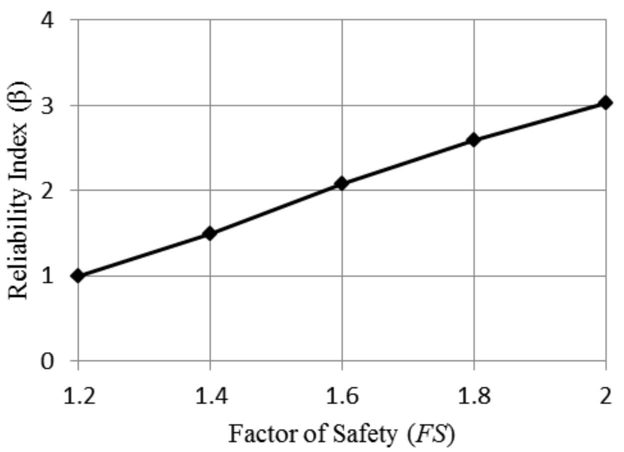

Figure 9. Variation of $FS-\beta$ for sliding mode of failure

Figures 9 and 10 denote the variation of reliability index with factor of safety for sliding and overturning modes of failure. From Figure 9 it is observed that the usually adopted factor of safety of 1.5 is not sufficient to get a reliability index of 3.0, which is desirable/required for engineering problems. In order to get a reliability index of 2.5, an FS of 1.8 is required whereas to get reliability index of 3.0, FS of 2.0, is required.

From Figure 10, it is clear that FS of 1.5 is sufficient to get the desired reliability index of 3.0 against overturning failure.

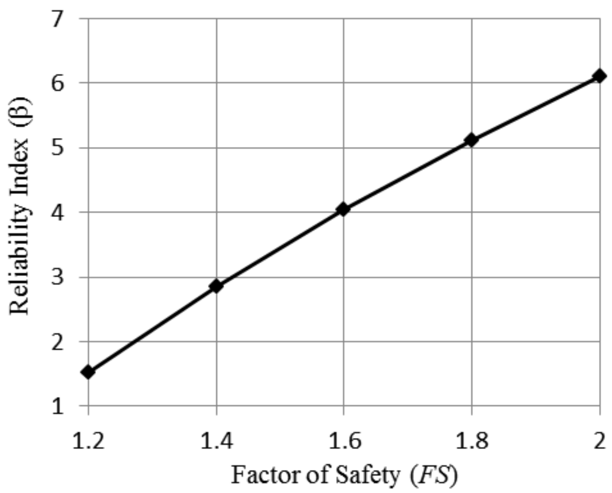

Figure 10. Variation of $FS-\beta$ for overturning mode of failure.

Among the structural modes of failure, the traditional deterministic *FS* of 1.5 against bending moment (positive) and shear failures is not able to achieve a target reliability index of 3.0.In order to get an idea on the *FS* required to get a target reliability index of 3.0, the plots showing the variation of reliability index with *FS* is generated for theses two modes of failure (Figure 11). From Figure 11 it is observed that an *FS* of 1.7 is required to get a reliability index of 3.0 for bending moment modes of failure (positive moment for heel slab) whereas a *FS* of 1.65 is required for shear mode of failure in heel slab.

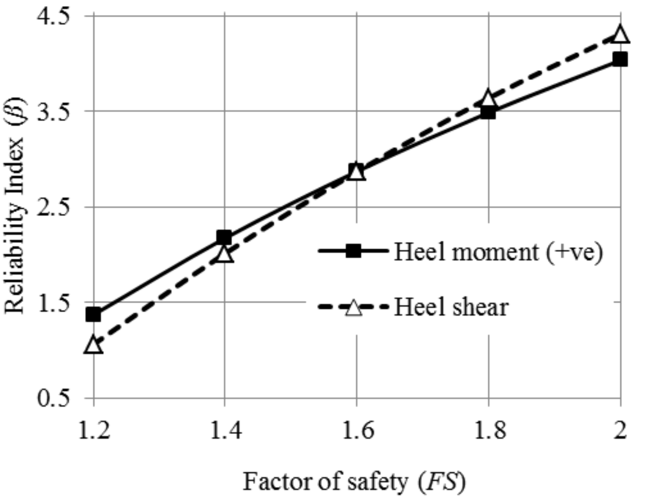

Figure 11. Variation of $FS - \beta$ for moment and shear mode of failure for heel slab

7 SUMMARY AND CONCLUSIONS

Uncertainties associated with the soil properties considerably affect the stability of the counterfort retaining walls. It is not wise to follow blindly the blanket 'factor of safety' approach without giving proper consideration to the uncertainties involved in the stability analysis of the walls. Probability based formulations offer a more rational way of evaluation of geotechnical stability analysis, in which probabilities of design failure can be assessed. Through this study, it is emphasized that for proper design of counterfort retaining walls, the designer has to address rationally the uncertainties involved in the design variables.

The following conclusions are drawn from this study:

1. Probability of failure is a more appropriate measure than the deterministic factor of safety (*FS*) for expressing geotechnical and structural stability of the retaining walls, since it considers uncertainties such as the variability in the parameters used for *FS* calculation.
2. The results of the reliability analysis obtained for the various geotechnical and structural failure modes of the counterfort retaining walls using the FORM, SORM and MCS methods are comparable.
3. Among the geotechnical modes of failure, sliding is the most critical one and a higher factor of safety (than the traditionally adopted *FS* of 1.5) is required. Among the structural modes of failure, positive moment failure in heel slab is the critical one and an *FS* of 1.7 is required to get a reliability index of 3.0.
4. Angle of internal friction of the backfill soil (ϕ) is the most sensitive random variable and has to be evaluated more realistically since a wide variation in reliability index occurs with the variation in the value of angle of internal friction.

NOMENCLATURE

A_{st}	area of steel
b	breadth of the slab
C_a	Rankine's active coefficient of active earth pressure
c	cohesion of soil
COV	coefficient of variation
D	total depth of slab
D_c	depth of counterfort slab
d	effective depth of slab
e	eccenticity
F_{cd}, F_{qd}, F_d	depth factors
F_{ci}, F_{qi}, F_i	load inclination factors
FS	factor of safety
FS_b	factor of safety against bearing
FS_s	factor of safety against sliding
f_{ck}	concrete strength
f_y	yield strength of steel
q_{max}	maximum pressure at the base slab
q_{min}	minimum pressure at the base slab
q_u	ultimate bearing capacity of the section
H	total height of reinforced retaining wall
h	depth of the foundation
h_1	height of wall without heel slab
L	width of base slab
L_c	clear spacing of counterforts
L_{cc}	centre to centre spacing of counterforts
L_h	length of heel slab
L_t	length of toe slab
M	moment at the section
N	number of simulations
N_c, N_q, N_γ	bearing capacity factors
N_f	number of simulation cycles when *g()* is less than zero
P_a	total active pressure on the back fill
P_5	uniformly distributed self-weight of the toe slab
P_f	probability of failure
PDF	probability density function
P_t	percentage of reinforcement

R	load
RBD	reliability based design
RF	shear capacity of the section
s	thickness of the stem at the top
S	resistance
SF	shear at the section due to loads
S_v	clear cover to reinforcement
V	shear force at the section
W	total weight of retained earth and wall
X	random variable
x_p	line of action of P_A from base of wall
x_w	line of action of W from heel
x_u	neutral axis depth
α	sensitivity vector by FORM
β	reliability index
β_{target}	target reliability index
γ_c	unit weight of concrete
γ_s	unit weight of backfill soil
μ	coefficient of friction
σ	standard deviation
ϕ	friction angle of backfill soil
ϕ_1	friction angle of soil beneath the base slab
ϕ	bar diameter
Φ(x)	normal cumulative distribution function at x
τ_c	shear strength of concrete

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