Buckling Reliability of Deteriorating Steel Beam Ends

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ABSTRACT: Deterioration of steel beam ends due to deicing media is a common problem in colder climates. A primary effect of this deterioration is a reduction in the buckling capacity of the steel beam due to thinning or loss of the web section above the bearing plate. Consequently, this can result in the need to post a reduced allowable load for a bridge possibly resulting in economic losses for nearby industries. It is the authors experience that when faced with this problem, a department of transportation structural analyst may make overly conservative assumptions. These assumptions may result in the posting or even closure of a bridge when it may not necessary. This paper presents the method and results of an alternate reliability-based damage assessment procedure using 1.) existing truckload data recorded at 42 weigh-in-motion (WIM) sites throughout the U.S. state of Michigan as the load and 2.) a detailed finite element model to determine the resistance of the section to buckling. A case study is presented for a selected bridge and varying levels of deterioration are modeled to examine the effect of web thinning on point-in-time (PIT) structural reliability index estimates, which are presented in chart format. Potential applications of such charts are qualitatively discussed including 1) reliability-based damage identification, 2) maintenance scheduling, and 3) integration into lifetime reliability models.
measured loads. Most studies have focused on the effect of time variant corrosion functions (e.g. exponential decay) on reliability with applications to inspection and maintenance (Kong et al. 2004). In addition, some studies use the design load to calculate reliability (Cheung et al. 2001). However, there can be significant discrepancy between the design live load and the observed/measured load (van de Lindt et al. 2002).

The work presented here differs from previous studies in several ways: 1.) Detailed structural models, i.e. finite element, are being applied to more accurately reflect the resistance side of the structural reliability formulation, 2.) measured load statistics from over 100 million trucks are used to model the live load effect rather than design loading, and finally 3.) the bias and coefficient of variation for the resistance are varied depending on the level of corrosion which is consistent with the observations of Moses et al. (1987). Of particular significance is the application of measured loads from 42 weigh-in-motion stations throughout Michigan, USA, divided by functional classification of roadway. It should be noted that, while the data is not perfect, it is considered significantly more accurate that weigh station data because the sensors are embedded in the pavement and the majority of the truck drivers do not know their vehicle is being weighed. Figure 2 shows the location of these WIM stations (www.mdot.gov).

2 STRUCTURAL RELIABILITY: BRIEF BACKGROUND

Structural reliability serves as the cornerstone of load and resistance factor design (LRFD) code calibration (Nowak 1995) essentially providing a measure by which to adjust the nominal design resistance of a structural component. For example, the LRFD bridge design code uses a target \( \beta \) of 3.5 (Nowak 1995, Nowak 1998) in an attempt to provide a desired safety reserve, albeit relative, in the bridge system. Reliability can also be used to help identify the severity of damage or whether or not a component should be repaired or replaced.

The probability of failure of a structure, \( p_f \), can be generalized as

\[
p_f = \text{Prob}(R < S) = \text{Prob}(Z < 0) = \text{Prob}(G(X) < 0)
\]  

(1)

where \( G(.) \) is the limit state function, \( X \) is the vector of loads, \( S \), and resistances, \( R \). If the probability distributions of the load and resistance are known then one may write the convolution integral

\[
p_f = \text{Prob}(R < S) = \int_{-\infty}^{\infty} F_R(x)f_S(x)dx
\]  

(2)

Numerous techniques ranging from Monte Carlo simulation (MCS) to nonlinear approximations are available to compute the failure probability and can be found in any reliability textbook (Madsen et al. 1986, Melchers 1999). The first order reliability method (FORM) was used in this study to compute the probability of failure, which is basically a linearization of the failure surface, and is a well accepted approximation for the majority of applications. Once the \( p_f \) is known the reliability index, \( \beta \), can be calculated as

\[
\beta = -\Phi^{-1}(p_f)
\]  

(3)

where \( \Phi^{-1}(.) \) is the inverse of the standard normal distribution function.

Figure 2. Weigh-in-motion (WIM) sensors in the state of Michigan (USA) shown by the red dots.
3 BUCKLING CAPACITY OF DETERIORATED STEEL BEAM ENDS

A simplified approach (used by some departments of transportation as an approximate solution strategy in lieu of time consuming finite element analysis) to determining this capacity is to assume the web is deteriorated over its full un-braced height regardless of the height of the damage, however as one might imagine, this is quite conservative. In the present study the buckling capacity of steel beam ends were determined by assuming that the deteriorated portion of the beam was nonexistent over a height equal only to the deteriorated section. The buckling load was estimated with finite element analysis (FEA) using solid parabolic tetrahedral elements with ten nodes and six degrees of freedom at each node. The solid tetrahedral elements were used in order to account for the complexity of the model and their ability to integrate well into a three-dimensional mesh.

Sensitivity analysis showed that the buckling capacity of a steel beam was not affected significantly by the shape of the deterioration other than the distance along the length of the beam and the depth of the corrosion (Kethu 2004). The corroded section is modelled by reducing the thickness of the web in the finite element model. Further details related to the finite element approach can be found in references (van de Lindt and Ahlborn 2004). The boundary conditions were modeled as fixed at the bottom and pinned at top which is consistent with previous analyses of this type. An eigenvalue buckling analysis was performed in order to find the buckling mode shape and from basic buckling theory, the lowest eigenvalue provided for the direct calculation of the buckling capacity. The applied load (shear) was distributed evenly over the bearing area as shown in Figure 3.

4 BRIDGE BEAMS: LOADS AND RESISTANCE

4.1 Live loads

Copy Live load models for bridges have been investigated statistically for only a few decades. Recently, Nowak (1995) proposed a live load model for highway bridges based on 9,250 surveyed WIM (weigh-in-motion) truck records. About that same time, a database on the order of tens of thousands of truck weights was used to calibrate the LRFD bridge code in a research report (Nowak 1999). In 2002, van de Lindt et al. (2002) investigated the design load in the state of Michigan using approximately forty thousand truck weights and axle spacing values from in and around the Detroit, MI area. All of these studies were aimed at the long-term load statistics, i.e. the statistical distribution of the \( n \)-year maximum load or load effect that could not be obtained directly from the surveyed data itself. A great deal of effort has been put toward the appropriate way to temporally “project” the short-term data to be representative of the \( n \)-year maximum load effect statistics. A method that uses the Gumbel distribution and is presented only briefly here (to be presented in its entirety in a paper forthcoming by the authors) was used to project a very large data set.

Weigh-in-motion (WIM) truck records from 42 locations in the U.S. state of Michigan arterial highway system (see Fig. 2), i.e. trunkline roadways, was used to develop the load effect (shear) for the steel beam ends. This data set consisted of axle weights and spacing of all trucks passing these 42 locations from 1997 to 2000, and during 2003. The data from 2001 and 2002 was not made available to the authors. There is a total of approximately 101 million truck records in this data set which was divided into four functional classifications of roadway according to U.S. National Bridge Inventory (NBI) classification. The functional classifications were identified as: 1.) FC01: Principal Arterial – Interstate Rural; 2.) FC02: Principal Arterial – Other – Rural; 3.) FC11: Principal Arterial – Interstate – Urban; and 4.) FC14: Other Principal Arterial – Urban. The data was grouped by functional classification so that each

![Figure 3. FE mesh and boundary conditions for the idealized model used in this study.](image-url)
data set contained tens of millions of truck weights and axle spacing. A database of load effects, i.e. moments and shears, was developed for bridges having various spans. The entire database was used for an LRFD code calibration of the design load in the state of Michigan. In the present study, data for one bridge span is extracted from the database for illustrative purposes.

Accurate projection of the WIM dataset representing 5-years of truckload data to represent the 75-year distribution of the maximum load requires a basic statistical temporal projection. Consider initially the general procedure applied to the prediction of a maximum in basic probability theory. Any field survey data, e.g. WIM dataset, for a random variable can be represented by an empirical or parametric statistical distribution function. This function could be given in the form of probability density function, \( f(x) \) (PDF) or cumulative distribution function, \( F(x) \), (CDF). When the statistical distribution of the maximum value of a random variable in the future is desired, one should first establish the relationship between the number of occurrences of the random event \( N \) with the future time period \( t \) (occurrence-time function). Functionally, this is

\[
N = g(t) \tag{4}
\]

The function \( g(t) \) may also come from field data collection. Then the corresponding CDF model for the maximum value of the random variable in future time period \( t \) can be written as

\[
F_{\text{max}}(x) = F(x)^{g(t)} \tag{5}
\]

Similarly, the PDF can be found by simply taking the derivative of the CDF in equation (5) with respect to \( x \), and is expressed as

\[
f_{\text{max}}(x) = \frac{dF_{\text{max}}(x)}{dx} = g(t)f(x)^{g(t)-1}f(x) \tag{6}
\]

With the PDF of the maximum value known, one can calculate the mean, standard deviation and higher statistical moments of the maximum value. If the PDF of the maximum value becomes analytically intractable, numerical integration can easily be applied. One should note that the mean and higher moments of the maximum value are only functions of the time period \( t \). The best model was found to be an Extreme Value (EV) Type I (Gumbel) distribution for all the projections to the statistical distribution of the 75 year maximum shear at the support. For the truck load data presented earlier, Table 1 presents the live load statistics in terms of the mean value and the coefficient of variation, \( COV \), defined as the ratio of the standard deviation to the mean) for a 21m (70ft) simply supported bridge.

### 4.2 Dead loads

The dead load for a bridge can be estimated several different ways. The first and most accurate for a specific bridge is to determine the weight directly from the bridge plans. Another method is to use a more generalized expression (see e.g. 7). The advantage of using a more generalized expression is that it is somewhat representative of a group of bridges with that span length, although introducing some amount of modelling uncertainty. However, this amount of uncertainty is approximately consistent with NCHRP studies (Moses et al. 1987, Nowak 1999). To do this, the dead load can be expressed in terms of the nominal live load, \( L_n \), the impact factor, \( I \), and the span as

\[
D = 0.0132(L_n + I) \times \text{span} \quad \tag{7}
\]

where \( \text{span} \) must be expressed in ft (1 ft = 0.302 m) for this dimensional relationship. Note that the dead load is assumed lognormal in this study consistent with recent bridge reliability studies in Michigan (van de Lindt et al., 2005). Impact factors have been measured and found to be approximately 1.1 for existing bridges. In the present study a value of 1.2 was selected as being slightly conservative compared with a very conservative AASHTO value of 1.3.

### 4.3 Resistance

Moses and Verma (1987) observed that, in general, as steel beam deterioration due to corrosion increases, the bias factor (defined as the ratio of the mean to nominal value) for the resistance decreases while the COV increases. Figure 4 shows a plot of the values assigned to the data base. The present authors propose to qualitatively calibrate those values to a percent reduction in buckling capacity that was calculated using extensive finite element analyses of

<table>
<thead>
<tr>
<th>Functional classification</th>
<th>Mean ( kN ) (kips)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC14</td>
<td>525.8 (118.3)</td>
<td>6.7</td>
</tr>
<tr>
<td>FC02</td>
<td>561.0 (126.2)</td>
<td>6.6</td>
</tr>
<tr>
<td>FC11</td>
<td>564.0 (126.9)</td>
<td>6.2</td>
</tr>
<tr>
<td>FC01</td>
<td>550.3 (123.8)</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 1. Mean and COV of the Gumbel-Distributed Live Load for the Example Bridge [W33 x 141; 21m (70 Ft) Span; 1.37 m (4.5 Ft) Beam spacing].
steel beam ends as previously discussed. The resistance data from finite element analyses was used to construct this correlation model. Figure 4 also presents a linear expression in terms of the percent capacity remaining (%C) for both the COV and bias. The mean resistance values were determined from the aforementioned finite element analysis, and the bias and COV are directly from Moses and Verma (1987).

Table 2. Mean, Bias, and COV for Other Random Variables in the Analysis.

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>Bias</th>
<th>COV(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live load effect, L</td>
<td>Gumbel</td>
<td>table 1</td>
<td>1.0</td>
<td>table 1</td>
</tr>
<tr>
<td>Dead load effect, D</td>
<td>Lognormal</td>
<td>0.0132(1+I)Ls</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>Girder Distribution</td>
<td>Lognormal</td>
<td>s/11</td>
<td>0.9</td>
<td>13</td>
</tr>
<tr>
<td>Dynamic impact factor, I</td>
<td>Lognormal</td>
<td>1.20</td>
<td>0.9</td>
<td>10</td>
</tr>
<tr>
<td>Resistance, R</td>
<td>Lognormal</td>
<td>from FEA Br*</td>
<td>COVr*</td>
<td></td>
</tr>
</tbody>
</table>

* Br=0.0028(%C)+0.83; COVr=-0.0023(%C)+0.34. Also see Figure 4 for graphical description of the resistance bias and COV.

Figure 4. Coefficient of Variation (COV) and Bias for the Buckling Resistance of a Deteriorated Steel Beam End.

5 ILLUSTRATIVE EXAMPLE

In order to demonstrate the approach proposed, let us investigate the effect of web deterioration, i.e. section reduction/loss, on the structural reliability index, β, of an existing bridge in the state of Michigan. It should be noted that the bridge does not actually have deterioration problems and has been selected to determine the span, beam spacing, and beam section dimensions only. In this example the bridge span is 21.3m (70 ft), the beams are spaced at 1.37m (4.5 ft), and the section is a W33 × 141 rolled section with a calculated buckling capacity of 1,500 kN (338 kips).

As previously discussed, FEA eigen-analysis was used to compute the remaining buckling capacity for the W33 × 141 beam having damage of various dimensions on one side and on both sides of the web. As mentioned, damage to the flange and the length of the damage along the longitudinal axis of the beam was found to have little effect on the buckling load during sensitivity analysis conducted as part of this study, so it was not modeled.

5.1 Reliability Analysis Results

Recall that the large WIM data set was divided by functional classification (FC) of roadway, hence the analysis was divided this way also. Figure 4 shows the change in the reliability index, β, as a function of the deterioration for FC 11 data. For example, if the average height of the deterioration (in terms of the percent of the un-braced web height) is 20%, the deterioration is only on one side of the web and averages about 4.8mm (3/16in) deep, then β = 4.25. This is illustrated by following the dashed lines of Figure 5. If the same level of deterioration is observed on both sides of the web the reliability index drops significantly to β = 2.7. Figure 6 presents relationships similar to Figure 5 for FC 01, FC 02, and FC 14.

Consider several applications for reliability charts such as those presented in Figures 4 and 5.

Table 3. Percent Buckling Capacity Remaining for a W33 × 141 Rolled Section Based on FEA using Solid Parabolic Tetrahedral Elements.

<table>
<thead>
<tr>
<th>Average height of damage</th>
<th>Average depth of damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>% unbraced depth</td>
<td>One side of web</td>
</tr>
<tr>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1.5</td>
<td>98</td>
</tr>
<tr>
<td>4.5</td>
<td>96</td>
</tr>
<tr>
<td>9.0</td>
<td>93</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
</tr>
<tr>
<td>27</td>
<td>86</td>
</tr>
</tbody>
</table>
5.1.1 Reliability-Based Damage Identification
In routine bridge inspection, situations arise in which one can measure (or estimate) the extent of deterioration, but it may difficult for the analyst to discern the effect of the damage on the system or component. This may be particularly true as the structural bridge community moves from allowable stress design (ASD) to LRFD. A basic reliability chart such as Figure 5 allows almost immediate identification of this effect if the dimensions/geometry of the damage to the beam web is known. For example, recall that the AASHTO LRFD code was calibrated to a reliability index of 3.5. Assuming one wanted to maintain this even if a section was damaged, then one could opt to target this reliability level as the point at which repairs or replacement should occur. In Figure 5, if the damage was approximately 4 to 5 mm deep on one side of the web it would have a reliability index to buckling over the bearing plate greater than 3.5 as long as the height of the corrosion remains less than 20% of the un-braced web height.

5.1.2 Maintenance Scheduling
Reliability-based maintenance scheduling generally requires the application of temporally varying deterioration models. These models can be updated as inspections occur. However, the deterioration models are generally based on a strength model and do not necessarily account for the geometry of the deteriorated section. The reliability charts presented here could integrate directly into maintenance scheduling approaches by providing the necessary point-in-time (PIT) reliability estimate to buckling load. Of course, numerous details would need to be addressed but conceptually the reliability information for particular damage geometries is present and suitable.

5.1.3 Integration into lifetime Reliability Models
Lifetime reliability models seek to represent the temporally varying reliability in order to incorporate the effects of one-time and recurring/scheduled maintenance and repairs, and in general, allow one to optimize resources and/or minimize losses for a structural system. Relatively complex limit state functions can be part of such models. In some cases the governing reliability index can be taken as the minimum of numerous limit state functions, essentially producing a lower bound estimate usually considered conservative. If a simple approach such as this is taken, i.e. identification of a governing failure mode based on reliability, then reliability charts such as those in Figure 5 can immediately provide the PIT estimate for inclusion in the analysis.
6 SUMMARY AND CONCLUSIONS

The results of a study whose objective was to develop reliability charts for buckling of deteriorated steel beam ends based on state-of-the-art measured truck load statistical modelling procedures was presented. Past studies have only investigated buckling reliability based on the design load. However, two issues make this inaccurate: 1.) designers generally exercise conservatism when selecting beams particularly with respect to bearing/shear loads, and 2.) truck loads are not equal to the design load and should be modelled as a realistic random variable.

It can be concluded based on this study that if a reasonable database of deteriorated beam end buckling capacity models are developed, site-specific (or at least FC-specific) reliability-based damage assessments are definitely possible. The amount of initial work would be significant. However, one should keep in mind that a database such as that could be developed for only the typical rolled sections used within a bridge inventory essentially eliminating a large portion of the repetitive work for structural analysts. Plate girders and specialty circumstances could still be handled on a case by case basis as is typically done now. Basic statistics as a function of span length is easily calculated and can be projected to the desired return period using the accurate projection technique discussed earlier, making integration into lifetime reliability models and maintenance scheduling a potential application.

REFERENCES